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**THERMODYNAMIC PROPERTIES OF GASES:
EQUATIONS DERIVED FROM THE
BEATTIE-BRIDGEMAN EQUATION OF STATE
ASSUMING VARIABLE SPECIFIC HEATS**

By

R. E. Randall
GDF, ARO, Inc.

August 1957

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SUMMARY

The Beattie-Bridgeman equation of state was used to develop the equations of several of the thermodynamic properties and flow process correction factors for gases. The increase in the specific heats due to the vibration of diatomic molecules was included by assuming the molecules to be perfect harmonic oscillators. Thermodynamic and flow process equations are theoretically developed. The particular flow processes investigated were isentropic expansions and flow through normal shock waves.

The calculation procedures for isentropic expansions and flow through normal shock waves are included in the appendix. Also included are the results of several calculations in which air was used as the media of flow. Graphs of the thermodynamic properties and Beattie-Bridgeman correction factors for air are included to provide a general picture of the effect of temperature and pressure.

NOMENCLATURE

a, b, c, A_0, B_0	Beattie-Bridgeman constants defined in appendix
a	Speed of sound, ft/sec
C	Specific heat, Btu/lb
e_1, e_2, e_3	Defined by Eq. (3)
E	Defined by Eq. (44)
F	Defined by Eq. (38)
$g_1, g_2, g_3, g_4, g_5, g_6$	Defined by Eq. (6)
G	Defined by Eq. (24)
h	Enthalpy, Btu/lb
I	Defined by Eq. (51)
J	Defined by Eq. (47)
k	$2_o\gamma/(\gamma-1)$
K	Defined by Eq. (45)
L	Defined by Eq. (61)
M	Mach number
p	Pressure, psia

q	Heat energy, Btu/lb
R	Gas Constant, (ft ³ lb) / (in. ² °R lb-mol)
S	Entropy, Btu/lb °R
T	Temperature, °R
u	Internal energy, Btu/lb
v	Volume, ft ³
V	Velocity, ft/sec
w	Mass flow, lb/ft ² sec
X	Defined by Eq. (58)
γ	Specific heat ratio
θ	Characteristic atomic vibration temperature in lowest energy state, °R
ρ	Density, lb/ft ³

PRESUBSCRIPTS

o	Perfect gas property
-----	----------------------

SUBSCRIPTS

c	Compression component
p	Constant pressure
r	Rotational component
t	Total (isentropic stagnation condition)
T	Constant temperature
v	Constant volume
tr	Translational component
vib	Vibrational component
∞	Infinite volume property (zero pressure)

INTRODUCTION

For wide ranges of pressures and temperatures, the perfect gas equation of state does not define accurately the relation between specific volume, pressure, and temperature of a gas. Furthermore, the ratio of specific heats changes considerably from the perfect gas values when the temperature and pressure of the gas differ widely from standard conditions. However, as long as the temperature remains between 300° and 700°R and the pressure remains between 0 and 30 psia during a thermodynamic process, the perfect gas relations provide relatively accurate values of the thermodynamic properties of gases.

The Gas Dynamics Facility, Arnold Engineering Development Center (GDF - AEDC) has wind tunnel circuits capable of producing pressure variations from near 0 to 2500 psia and temperature variations from about 70° to about 2000°R. Since test results based on the perfect gas relations and constant specific heats are apt to be in error, a method of correction is needed.

The Beattie-Bridgeman equation of state and the values of the constants for several gases were published in 1928 (see Ref. 1). The purpose of the equation was to provide a means of calculating more accurate values of the thermodynamic properties of actual gases. However, the thermodynamic relations developed from this equation are extremely complicated and cumbersome when compared with the relations developed from the perfect-gas equation of state. The additional work involved is not ordinarily justified by the increased accuracy of the calculations; therefore, the equation has never been used extensively.

The need for greater accuracy in research work has caused a renewed effort to develop methods of calculating gas properties with an accuracy in keeping with the precision of experimental instrumentation. This work is submitted as a step toward this end, and with an immediate objective of permitting a more accurate calibration of the GDF wind tunnels.

This report is an elaboration of the work reported in Ref. 2 and makes several changes such as: (1) the use of engineering units, (2) the addition of two more terms to the infinite series expressing density ratio, (3) the use of more recently determined values of the characteristic temperature of vibration for molecular nitrogen and oxygen, (4) the use of standard gas mixture proportions for air as reported in Ref. 3, and (5) the inclusion of two specific methods of solving flows through normal shock waves.

DEVELOPMENT OF THE THERMODYNAMIC EQUATIONS

DENSITY

The Beattie-Bridgeman equation of state for an imperfect gas is (see Ref. 1)

$$p = \frac{RT}{v^2} \left(1 - \frac{c}{vT^3} \right) \left[v + B_o \left(1 - \frac{b}{v} \right) \right] - \frac{A_o}{v^2} \left(1 - \frac{a}{v} \right) \quad (1)$$

Expanding Eq. (1) and collecting terms we can obtain

$$p = \frac{RT}{v} + \left(B_o - \frac{A_o}{RT} - \frac{c}{T^3} \right) \frac{RT}{v^2} + \left(\frac{A_o a}{RT} - B_o b - \frac{B_o c}{T^3} \right) \frac{RT}{v^3} + \left(\frac{B_o b c}{T^3} \right) \frac{RT}{v^4} \quad (2)$$

Letting

$$\left. \begin{aligned} e_1 &= B_o - \frac{A_o}{RT} - \frac{c}{T^3} \\ e_2 &= \frac{A_o a}{RT} - B_o b - \frac{B_o c}{T^3} \\ e_3 &= \frac{B_o b c}{T^3} \end{aligned} \right\} \quad (3)$$

and using the relation

$$\rho = \frac{1}{v}$$

we can write Eq. (2) in the form

$$p = \rho RT (1 + e_1 \rho + e_2 \rho^2 + e_3 \rho^3) = \rho (1 + e_1 \rho + e_2 \rho^2 + e_3 \rho^3) \left(1 + \frac{e_1 \rho}{RT} + \frac{e_2 \rho^2}{RT} + \frac{e_3 \rho^3}{RT} + \dots \right) \quad (4)$$

Let ρ be represented by an infinite series of the form

$$\rho = \frac{P}{RT} (1 + g_1 P + g_2 P^2 + g_3 P^3 + \dots) \quad (5)$$

Obtaining successive powers of Eq. (4), substituting in Eq. (5) and solving for the values of g_1 through g_6 gives the following relations:

$$\begin{aligned}
g_1 &= -\frac{e_1}{RT} \\
g_2 &= \frac{2e_1^2 - e_2}{(RT)^2} \\
g_3 &= \frac{5e_1(e_2 - e_1^2) - e_3}{(RT)^3} \\
g_4 &= \frac{6e_1e_3 + 3e_2^2 + 7e_1^2(2e_1^2 - 3e_2)}{(RT)^4} \\
g_5 &= \frac{7[e_2e_3 - 4e_1(e_2^2 + e_1e_3) + 6e_1^3(2e_2 - e_1^2)]}{(RT)^5} \\
g_6 &= \frac{6[10e_1^2(3e_2^2 + 2e_1e_3) + 11e_1^4(2e_1^2 - 5e_2) - 2e_2^3] + e_3(4e_3 - 73e_1e_2)}{(RT)^6}
\end{aligned} \tag{6}$$

Substituting in Eq. (5) the perfect gas relation

$$\rho = \frac{p}{RT}$$

we obtain the density ratio

$$\frac{\rho}{\rho_0} = 1 + g_1 p + g_2 p^2 + g_3 p^3 + \dots \tag{7}$$

SPECIFIC HEAT

The internal energy of a gas consists of the kinetic energy of the molecules and the atoms within the molecules, and the potential energy of the molecules. When energy is added to a gas, each of these internal energy factors takes up its portion according to the characteristics of the gas involved.

The kinetic energy of the molecules is dependent upon the number of significant degrees of freedom of translation and rotation, and the temperature of the gas. Most gas molecules take up an amount of energy very nearly equal to $1/2 RT$ per mol of gas for each significant degree of freedom. Translation takes place along the three axes of space for all temperatures, even approaching absolute zero. Thus, the translation of the molecules provides three degrees of freedom. Rotation about the three axes of space begins at some finite temperature above absolute zero, and the rotational degrees of freedom become fully activated at a slightly higher temperature. The number of axes about

which rotation is significant and the temperature above which full activation is attained are dependent upon the particular gas involved. Once a particular gas is selected and the number of significant degrees of freedom determined, the molecular kinetic energy becomes a function of the temperature of the gas alone.

The atomic kinetic energy is dependent upon the characteristics of the individual atoms, the number of atoms involved, and the temperature of the gas. This energy is held by the moving atoms as they oscillate with respect to one another within the molecule and is called the vibration energy.* As with the molecular kinetic energy, once the particular gas is selected and the vibration characteristics determined, the atomic kinetic energy becomes a function of the temperature of the gas alone.

The molecular potential energy is the energy required to force the molecules into close association with one another. In any real gas this energy is a function of any two of the variables of state, p , v , and T . The perfect gas equation of state and its accompanying thermodynamic relations consider only the molecular kinetic energy; they neglect both the atomic kinetic energy and the molecular potential energy. The Beattie-Bridgeman equation of state and its accompanying thermodynamic relations are based upon a consideration of the molecular kinetic and potential energy. If the atoms involved approach perfect harmonic oscillators, the atomic kinetic energy can be easily accounted for in either the perfect gas or Beattie-Bridgeman thermodynamic relations.

The rotation of monatomic molecules does not represent a significant amount of energy since its moment of inertia is negligible about the three axes of space. Since there is only one atom present in the molecule, there can be no atomic vibration. Thus, the internal energy of a monatomic gas consists of only the molecular kinetic energy of translation and the molecular potential energy. From the preceding discussion we can see that the internal energy of a monatomic gas should be about $3/2 RT$ per mol when the molecular potential energy is negligible.

The diatomic molecule presents a problem which is considerably different from the monatomic molecule. The three degrees of freedom

* The electronic, ionization, and dissociation energies will not be discussed since they become significant only at temperatures above the range considered in this work.

of translation remain the same. However, considering the molecule to have a dumb-bell like form, where the atoms are spaced a short distance apart and held together by a rigid bar, the moments of inertia about the three axes of space are no longer negligible. Visualizing one axis coinciding with the bar connecting the two atoms, we see that the moment of inertia about this axis is negligible when compared with the moments of inertia about the other two axes of space. From this analysis we see that for diatomic molecules two degrees of freedom of rotation must be added to the three degrees of freedom of translation. Thus, the internal energy of a diatomic gas should be about $5/2 RT$ when the molecular potential energy and the atomic kinetic energy are negligible.

The potential energy of a gas approaches zero as the pressure approaches zero (or the volume becomes infinite) regardless of the gas temperature. Thus, one can conceive of a hypothetical expansion of a gas by a constant-temperature process until the volume becomes infinite. Then the only internal energy remaining in the gas is the kinetic energy of the molecules and the kinetic energy of the atoms. Although not rigorously additive, a sufficient degree of approximation can be obtained by separating the kinetic energy into the sum of its three components as follows:

$$u_{\infty} = u_{tr} + u_r + u_{vib} \quad (8)$$

(u_r and u_{vib} are not independent; see Ref. 4).

Since the internal energy of a perfect gas considers the translational and rotational energy, we can write Eq. (8) as

$$u_{\infty} = u + u_{vib} \quad (9)$$

The specific heat of a gas is given by the thermodynamic relation

$$C = \frac{\delta q}{dT} \quad (10)$$

where

$$\delta q = du + pdv \quad (11)$$

To determine the specific heat due to vibration, the pressure is assumed zero and we can write from Eqs. (10) and (11)

$$C_{vib} = \frac{du_{vib}}{dT} \quad (12)$$

Integration gives

$$\int_0^T C_{vib} dT = \int_0^{u_{vib}} du_{vib} \quad (13)$$

or

$$u_{\text{vib}} = \int_0^T C_{\text{vib}} dT \quad (14)$$

Both C_p and C_v will be increased by C_{vib} when atomic vibrations are considered.

The internal energy of the gas at infinite volume is given by Eq. (8) as u_{∞} . To obtain the total internal energy one can assume a hypothetical constant temperature compression of the gas to normal pressure; then the total internal energy of the gas is the sum of the infinite volume internal energy and the energy of compression. Thus, we can write

$$u = u_{\infty} + u_c = u_{\infty} + u_{\text{vib}} + u_c \quad (15)$$

where u_c is a function of any two of the state variables p , v , and T . Considering it to be a function of v and T we can write

$$u_c = u(v, T) \quad (16)$$

from which

$$du_c = \left(\frac{\partial u}{\partial v} \right)_T dv + \left(\frac{\partial u}{\partial T} \right)_v dT \quad (17)$$

Since a constant temperature process is to be used for the compression, Eq. (17) can be written

$$du_c = \left(\frac{\partial u}{\partial v} \right)_T dv \quad (18)$$

Integrating Eq. (18) we have

$$u_c = \int_{\infty}^v \left(\frac{\partial u}{\partial v} \right)_T dv \quad (19)$$

When the thermodynamic identity $\left(\frac{\partial u}{\partial v} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_v - p$ is used,

Eq. (19) becomes

$$u_c = \int_{\infty}^v T \left(\frac{\partial p}{\partial T} \right)_v dv - \int_{\infty}^v p dv \quad (20)$$

When we expand Eq. (2), differentiate with respect to T holding v constant, and substitute into Eq. (20), we get

$$u_c = \int_{\infty}^v \left[\frac{A_0}{v^2} + \frac{3Rc}{v^2 T^2} - \frac{A_0 a}{v^3} + \frac{3B_0 Rc}{v^3 T^2} - \frac{3B_0 b Rc}{v^4 T^2} \right] dv$$

Integration and the substitution of $\rho = \frac{1}{v}$ gives

$$u_c = R \left[-\rho \left(\frac{A_o}{R} + \frac{3c}{T^2} \right) - \rho^2 \left(\frac{3B_{oc}}{2T^2} - \frac{A_{oa}}{2R} \right) + \rho^3 \left(\frac{B_{obc}}{T^2} \right) \right] \quad (21)$$

Using Eqs. (14), (21), and the perfect gas relation ${}_o u = {}_o C_v T$ in Eq. (15) produces the equation for the total internal energy of a Beattie-Bridgeman gas considering the contribution of the atomic vibrations. This equation is

$$u = T \left\{ {}_o C_v + \frac{1}{T} \int_0^T C_{vib} dT + R \left[-\rho \left(\frac{A_o}{RT} + \frac{3c}{T^3} \right) - \rho^2 \left(\frac{3B_{oc}}{2T^3} - \frac{A_{oa}}{2RT} \right) + \rho^3 \left(\frac{B_{obc}}{T^3} \right) \right] \right\} \quad (22)$$

If we use the perfect gas relation

$$R = {}_o C_v ({}_o \gamma - 1) \quad (23)$$

and let

$$G = \frac{1}{RT} \int_0^T C_{vib} dT \quad (24)$$

Eq. (22) becomes

$$u = {}_o C_v T \left\{ 1 + ({}_o \gamma - 1) \left[G - \rho \left(\frac{A_o}{RT} + \frac{3c}{T^3} \right) - \rho^2 \left(\frac{3B_{oc}}{2T^3} - \frac{A_{oa}}{2RT} \right) + \rho^3 \left(\frac{B_{obc}}{T^3} \right) \right] \right\} \quad (25)$$

Using Eqs. (10) and (11) to obtain the specific heat for a constant volume process gives

$$C_v = \left(\frac{du}{dT} \right)_v \quad (26)$$

Substituting Eq. (21) into Eq. (15) and differentiating with respect to T with v constant produces

$$C_v = \left(\frac{du}{dT} \right)_v = \frac{d{}_o u}{dT} + \frac{du_{vib}}{dT} + \frac{6Rc}{T^3} \rho \left(1 + \frac{B_o}{2} \rho - \frac{B_{ob}}{3} \rho^2 \right) \quad (27)$$

Using Eq. (12) and the perfect gas relation

$${}_o C_v = \frac{d{}_o u}{dT}$$

we have the specific heat at constant volume for a Beattie-Bridgeman gas with the contribution of the atomic vibrations considered

$$C_v = {}_o C_v + C_{vib} + \frac{6Rc}{T^3} \rho \left(1 + \frac{B_o}{2} \rho - \frac{B_{ob}}{3} \rho^2 \right) \quad (28)$$

The specific heat at constant pressure can be determined from the thermodynamic identity

$$C_p - C_v = -T \frac{(\partial p / \partial T)_v^2}{(\partial p / \partial v)_T} \quad (29)$$

Obtaining $(\partial p / \partial T)_v^2$ and $(\partial p / \partial v)_T$ from Eq. (2) we find

$$C_p - C_v = R \left\{ \frac{\left[\left(1 + \frac{2c}{T^3} \rho \right) \left(1 + B_o \rho - B_o b \rho^2 \right) \right]^2}{1 + 2e_1 \rho + 3e_2 \rho^2 + 4e_3 \rho^3} \right\} \quad (30)$$

With the equations for C_v and C_p , the ratio of specific heats is determined in the usual way as

$$\gamma = \frac{C_p}{C_v}$$

ENTHALPY

The enthalpy of a gas is defined by the equation

$$h = u + \frac{p}{\rho}$$

If we rewrite Eq. (2) in the form

$$\frac{p}{\rho} = {}_oC_v T ({}_o\gamma - 1) \left[1 + \rho \left(B_o - \frac{A_o}{RT} - \frac{c}{T^3} \right) + \rho^2 \left(\frac{A_o a}{RT} - B_o b - \frac{B_o c}{T^3} \right) + \rho^3 \left(\frac{B_o b c}{T^3} \right) \right]$$

and add Eq. (25), we get the enthalpy relation for a Beattie-Bridgeman gas with the contribution of atomic vibrations considered:

$$h = {}_oC_v T \left\{ {}_o\gamma + ({}_o\gamma - 1) \left[G + \rho \left(B_o - \frac{2A_o}{RT} - \frac{4c}{T^3} \right) + \rho^2 \left(\frac{3A_o a}{2RT} - \frac{5B_o c}{2T^3} - B_o b \right) + \rho^3 \left(\frac{2B_o b c}{T^3} \right) \right] \right\}$$

Using the two perfect gas relations

$$\begin{aligned} {}_oC_v &= \frac{{}_oC_p}{{}_o\gamma} \\ {}_oh &= {}_oC_p T \end{aligned} \quad (31)$$

we obtain the ratio of the Beattie-Bridgeman enthalpy to the perfect gas enthalpy:

$$J = \frac{h}{{}_oh} = 1 + \frac{({}_o\gamma - 1)}{{}_o\gamma} \left[G + \rho \left(B_o - \frac{2A_o}{RT} - \frac{4c}{T^3} \right) + \rho^2 \left(\frac{3A_o a}{2RT} - \frac{5B_o c}{2T^3} - B_o b \right) + \rho^3 \left(\frac{2B_o b c}{T^3} \right) \right] \quad (32)$$

ENTROPY

Entropy is a state variable the same as p , v , T , and u . Therefore, we may obtain the change in entropy between any two gas conditions by proceeding from one condition to the other by any desired method or path.

The thermodynamic relation for entropy is

$$dS = \frac{C_v}{T} dT + \left(\frac{\partial p}{\partial T} \right)_v dv$$

Integrating between point 1 and o, we get

$$\int_1^o dS = \int_1^o \frac{C_v}{T} dT + \int_1^o \left(\frac{\partial p}{\partial T} \right)_v dv$$

Holding T constant at T_1 and evaluating the change in entropy involved when v changes from v_1 to v_o and designating this change as $(S_o - S_1)_2$ we have

$$(S_o - S_1)_2 = \int_{v_1(T=T_1)}^{v_o} \left(\frac{\partial p}{\partial T} \right)_v dv \quad (33)$$

Next, holding v constant at v_o , evaluating the change in entropy involved when T changes from T_1 to T_o and designating this change as $(S_o - S_1)_1$ we have

$$(S_o - S_1)_1 = \int_{T_1(v=v_o)}^{T_o} \frac{C_v}{T} dT \quad (34)$$

Finally, the total change in entropy when proceeding from point 1 to point o is given by adding Eqs. (33) and (34):

$$S_o - S_1 = (S_o - S_1)_1 + (S_o - S_1)_2 = \int_{T_1(v=v_o)}^{T_o} \frac{C_v}{T} dT + \int_{v_1(T=T_1)}^{v_o} \left(\frac{\partial p}{\partial T} \right)_v dv$$

Replacing C_v in Eq. (34) by Eq. (28), we get

$$(S_o - S_1)_1 = \int_{T_1}^{T_o} \frac{oC_v}{T} dT + \int_{T_1}^{T_o} \frac{C_{vib}}{T} dT + \int_{T_1}^{T_o} \frac{6Rc}{T^4} \rho \left(1 + \rho \frac{B_o}{2} - \rho^2 \frac{B_o b}{3} \right) dT \quad (35)$$

Using Eq. (23) and evaluating the first integral of Eq. (35) gives

$$\frac{R}{o\gamma - 1} \int_{T_1}^{T_o} \frac{dT}{T} = R \ln \left(\frac{T_o}{T_1} \right)^{\frac{1}{o\gamma - 1}}$$

Let the second integral of Eq. (35) be defined as

$$\int_{T_1}^{T_o} \frac{C_{vib}}{T} dT = R \ln \frac{E_o}{E_1} \quad (36)$$

Evaluating the third integral of Eq. (35) we have

$$R \int_{T_1(\rho=\rho_o)}^{T_o} \frac{6c}{T^4} \rho \left(1 + \rho \frac{B_o}{2} - \rho^2 \frac{B_o b}{3} \right) dT = R \left[c\rho_o \left(2 + \rho_o B_o - \rho_o^2 \frac{2B_o b}{3} \right) \left(\frac{1}{T_1^3} - \frac{1}{T_o^3} \right) \right]$$

Replacing these three integrals in Eq. (35) we have

$$(S_o - S_1)_1 = R \left\{ \ln \left[\left(\frac{T_o}{T_1} \right)^{\frac{1}{\gamma-1}} \left(\frac{E_o}{E_1} \right) \right] + c\rho_o \left(2 + \rho_o B_o - \rho_o^2 \frac{2B_o b}{3} \right) \left(\frac{1}{T_1^3} - \frac{1}{T_o^3} \right) \right\} \quad (37)$$

Obtaining $\left(\frac{\partial p}{\partial T} \right)_v$ from Eq. (2) and substituting into Eq. (33) gives

$$(S_o - S_1)_2 = \int_{v_1(T=T_1)}^{v_o} R \left[\frac{1}{v} + \frac{1}{v^2} \left(B_o + \frac{2c}{T^3} \right) + \frac{1}{v^3} \left(\frac{2B_o c}{T^3} - B_o b \right) - \frac{1}{v^4} \left(\frac{2B_o b c}{T^3} \right) \right] dv$$

Integrating, replacing $\frac{1}{v}$ by ρ and adding to Eq. (37) gives

$$S_o - S_1 = R \left\{ \ln \left[\left(\frac{T_o}{T_1} \right)^{\frac{1}{\gamma-1}} \frac{E_o}{E_1} \frac{\rho_1}{\rho_o} \right] - \rho_o \left(B_o + \frac{2c}{T_o^3} \right) - \rho_o^2 \left(\frac{B_o c}{T_o^3} - \frac{B_o b}{2} \right) \right. \\ \left. + \rho_o^3 \frac{2B_o b c}{3 T_o^3} + \rho_1 \left(B_o + \frac{2c}{T_1^3} \right) + \rho_1^2 \left(\frac{B_o c}{T_1^3} - \frac{B_o b}{2} \right) - \rho_1^3 \frac{2B_o b c}{3 T_1^3} \right\}$$

Letting

$$\ln F = \rho \left(B_o + \frac{2c}{T^3} \right) + \rho^2 \left(\frac{B_o c}{T^3} - \frac{B_o b}{2} \right) - \rho^3 \frac{2B_o b c}{3 T^3} \quad (38)$$

we have the change in entropy between condition *o* and condition *1* for a Beattie-Bridgeman gas considering the contribution of atomic vibrations:

$$S_o - S_1 = R \ln \left[\left(\frac{T_o}{T_1} \right)^{\frac{1}{\gamma-1}} \frac{E_o}{E_1} \frac{\rho_1}{\rho_o} \frac{F_1}{F_o} \right] \quad (39)$$

DIATOMIC VIBRATION CORRECTIONS

Diatomic molecules have only one normal mode of vibration. If the vibration characteristics of the molecule approach the characteristics of a perfect harmonic oscillator, the vibration component of specific heat is given as (see Ref. 5)

$$C_{\text{vib}} = R \left[\frac{\theta/2T}{\sinh(\theta/2T)} \right]^2 = R \frac{(\theta/T)^2 \exp \theta/T}{(\exp \theta/T - 1)^2} \quad (40)$$

When C_{vib} in Eq. (14) is replaced by Eq. (40), it becomes

$$u_{\text{vib}} = R \int_0^T \left[\frac{\theta/2T}{\sinh(\theta/2T)} \right]^2 dT$$

To integrate let

$$y = \frac{\theta}{2T}$$

Then

$$dy = -\frac{\theta}{2T^2} dT$$

Note that as $T \rightarrow 0$, $y \rightarrow \infty$ and as $T \rightarrow T$, $y \rightarrow y$. Thus,

$$u_{\text{vib}} = -R \frac{\theta}{2} \int_{\infty}^y \frac{dy}{\sinh^2 y}$$

Dividing through by RT we have

$$\frac{u_{\text{vib}}}{RT} = G = \frac{1}{RT} \int_0^T C_{\text{vib}} dT = -y \int_{\infty}^y \frac{dy}{\sinh^2 y} \quad (41)$$

which, when integrated, yields

$$G = \frac{2y}{\exp(2y) - 1} \quad (42)$$

Equation (42) gives the contribution of the vibration of the atoms to the internal energy of a diatomic gas provided the vibration characteristics of the molecule approach the characteristics of a perfect harmonic oscillator.

Evaluating Eq. (36) in a similar manner, we obtain

$$\int_{T_1}^{T_0} \frac{C_{\text{vib}}}{T} dT = \int_0^{T_0} \frac{C_{\text{vib}}}{T} dT - \int_0^{T_1} \frac{C_{\text{vib}}}{T} dT = R \ln \frac{E_0}{E_1} \quad (43)$$

We find the solution of the first right hand integral to be

$$\int_0^T \frac{C_{\text{vib}}}{T} dT = -R \int_{\infty}^y \frac{y dy}{\sinh^2 y} = R \left[y \coth y - \ln |\sinh y| \right]_{\infty}^y$$

Since we are dealing with only real values of y , $\sinh y$ is always positive. Thus, $|\sinh y| = \sinh y$. Substituting the limits we have

$$R \ln E = \int_0^T \frac{C_{vib}}{T} dT = R \left[y \coth y - \ln(2 \sinh y) \right] \quad (44)$$

Solving Eq. (44) for E gives

$$E = \frac{\exp[2y/(\exp 2y - 1)]}{1 - \exp(-2y)}$$

DEVELOPMENT OF THE FLOW PROCESS EQUATIONS

ISENTROPIC FLOW

Since an isentropic process involves no change in entropy, we may equate the right side of Eq. (39) to zero.

Thus,

$$0 = \ln \left[\left(\frac{T}{T_1} \right)^{\frac{1}{\sigma\gamma-1}} \frac{E}{E_1} \frac{\rho_1}{\rho} \frac{F_1}{F} \right]$$

or

$$1 = \left(\frac{T}{T_1} \right)^{\frac{1}{\sigma\gamma-1}} \frac{E/F}{E_1/F_1} \frac{1/\rho}{1/\rho_1}$$

Multiplying both sides by p/p_1 and rearranging

$$\frac{p}{p_1} = \left(\frac{T}{T_1} \right)^{\frac{\sigma\gamma}{\sigma\gamma-1}} \frac{E/F}{E_1/F_1} \frac{p/\rho RT}{p_1/\rho_1 R T_1}$$

Using Eq. (4) and letting

$$K = \frac{E}{F} (1 + e_1 \rho + e_2 \rho^2 + e_3 \rho^3) \quad (45)$$

we obtain

$$\frac{p}{p_1} = \left(\frac{T}{T_1} \right)^{\frac{\sigma\gamma}{\sigma\gamma-1}} \frac{K}{K_1} \quad (46)$$

Let

$$J = \frac{h}{\sigma h} \quad (47)$$

as defined by Eq. (32); then we can obtain

$$h = {}_oC_p T_t \quad (48)$$

Using the thermodynamic relation

$$V^2 = 2(h_t - h) \quad (49)$$

and replacing h by Eq. (48) we have

$$V^2 = 2{}_oC_p T \left(\frac{T_t J_t}{T} - J \right) \quad (50)$$

The thermodynamic relation for the speed of sound is

$$a^2 = \gamma \left(\frac{\partial p}{\partial \rho} \right)_T$$

Differentiating Eq. (4) and substituting in the above equation we get

$$a^2 = \gamma RT [1 + 2e_1 \rho + 3e_2 \rho^2 + 4e_3 \rho^3]$$

Letting

$$I = \frac{\gamma}{o\gamma} [1 + 2e_1 \rho + 3e_2 \rho^2 + 4e_3 \rho^3] \quad (51)$$

we obtain

$$a^2 = I {}_o\gamma RT = I ({}_o a^2) \quad (52)$$

Dividing Eq. (50) by Eq. (52) and using Eqs. (23) and (31) gives

$$M^2 = \frac{V^2}{a^2} = \frac{2 \left(\frac{T_t J_t}{T} - J \right)}{(o\gamma - 1) I} \quad (53)$$

Solving for $\frac{T_t}{T}$ produces

$$\frac{T_t}{T} = \frac{\frac{o\gamma - 1}{2} IM^2 + J}{J_t} \quad (54)$$

or solving for T we have

$$T = \frac{T_t J_t}{\frac{o\gamma - 1}{2} IM^2 + J} \quad (55)$$

Rewriting Eq. (46) in terms of stagnation quantities and substituting Eq. (54) gives

$$\frac{p_t}{p} = \left(\frac{\frac{o\gamma - 1}{2} IM^2 + J}{J_t} \right)^{\frac{o\gamma}{o\gamma - 1}} \frac{K_t}{K} \quad (56)$$

Solving Eq. (56) for M we obtain

$$M = \sqrt{\frac{J_t \left(\frac{p_t K}{p K_t} \right)^{\frac{o\gamma-1}{o\gamma}} - J}{\frac{o\gamma-1}{2} I}} \quad (57)$$

Substituting Eq. (48) into Eq. (49), dividing by Eq. (52) and using the relations in Eqs. (23) and (31) and $M^2 = \frac{V^2}{a^2}$ we find that

$$\frac{2h_t}{V^2} = \frac{M^2 + \frac{2}{o\gamma-1} \frac{J}{I}}{M^2}$$

Letting

$$X = \frac{V^2}{2h_t} = \frac{M^2}{M^2 + \frac{2}{o\gamma-1} \frac{J}{I}} \quad (58)$$

and solving for M^2 we get the equation

$$M^2 = \frac{2}{o\gamma-1} \frac{J}{I} \frac{X}{1-X} \quad (59)$$

Replacing M^2 in Eq. (55) by Eq. (59) we obtain the solution

$$T = \frac{T_t J_t}{J} (1-X) \quad (60)$$

By substituting Eq. (48) into Eq. (49), multiplying both sides by $\frac{p}{\rho V}$, and using Eqs. (23) and (31) it is found that

$$\frac{p}{\rho V} = \frac{p}{\rho R T J} \frac{o\gamma-1}{2o\gamma} \left(\frac{2h_t - V^2}{V} \right)$$

Letting

$$L = \frac{p}{\rho R T J} \quad (61)$$

and

$$k = \frac{2o\gamma}{o\gamma-1}$$

we obtain the result,

$$\frac{p}{\rho V} = \frac{L}{k} \left(\frac{2h_t - V^2}{V} \right) \quad (62)$$

$$\frac{p}{\rho V} = \frac{L}{k} \left(\frac{2h_t}{V} - V \right)$$

FLOW THROUGH NORMAL SHOCK WAVES

In order to determine the relation between the variables of state on the forward and aft side of a normal shock wave it is necessary to apply three equations to the flow through the shock wave:

Conservation of Mass

$$w = \rho_1 V_1 = \rho_2 V_2 \quad (63)$$

Conservation of Momentum

$$\frac{p_2}{w} - \frac{p_1}{w} = V_1 - V_2 \quad (64)$$

Conservation of Energy

$$h_{t1} = h_{t2} \quad (65)$$

NOTE: The subscripts 1 and 2 apply, respectively, to conditions forward and aft of the shock wave.

Substituting Eq. (63) into Eq. (64) and using Eq. (62) gives

$$L_2 \left(\frac{2h_{t2}}{V_1^2} - \frac{V_1}{V_2} - \frac{V_2}{V_1} \right) - L_1 \left(\frac{2h_{t1}}{V_1^2} - 1 \right) = k \left(1 - \frac{V_2}{V_1} \right)$$

From Eqs. (63), (58), and (65) it follows that

$$k \left(1 - \frac{\rho_1}{\rho_2} \right) = L_2 \left(\frac{1}{X_1} \frac{\rho_2}{\rho_1} - \frac{\rho_1}{\rho_2} \right) - L_1 \left(\frac{1}{X_1} - 1 \right)$$

Multiplying through by $X_1 \frac{\rho_1}{\rho_2}$ and collecting terms, we get

$$\left(\frac{\rho_1}{\rho_2} \right)^2 \left[X_1 (L_2 - k) \right] + \frac{\rho_1}{\rho_2} \left[X_1 (k - L_1) + L_1 \right] - L_2 = 0 \quad (66)$$

From Eqs. (58), (65), and (63) we can establish that

$$\frac{X_1}{X_2} = \left(\frac{\rho_2}{\rho_1} \right)^2 \quad (67)$$

Also, from Eqs. (48) and (65) we find

$$T_{t1} J_{t1} = T_{t2} J_{t2} \quad (68)$$

Let Eq. (53) be written in the form

$$\frac{M_2^2}{M_1^2} = \frac{I_1 T_1 (J_{t2} T_{t2} - J_2 T_2)}{I_2 T_2 (J_{t1} T_{t1} - J_1 T_1)}$$

By using Eqs. (68), (60), and (67) it follows that

$$\frac{M_2^2}{M_1^2} = \frac{I_1 T_1}{I_2 T_2} \left(\frac{\rho_1}{\rho_2} \right)^2 \quad (69)$$

Write Eq. (62) in the form

$$\frac{p_2}{p_1} = \frac{\rho_2 L_2 (1 - V_2^2 / 2h_{t2})}{\rho_1 L_1 (1 - V_1^2 / 2h_{t1})}$$

in which we substitute Eq. (58) to get

$$\frac{p_2}{p_1} = \frac{\rho_2 L_2 (1 - X_2)}{\rho_1 L_1 (1 - X_1)} \quad (70)$$

Substituting Eq. (63) into Eq. (64), dividing through by two times Eq. (48) and using Eq. (65), we obtain

$$\frac{p_2}{2_o C_p T_{t1} J_{t1}} + \frac{\rho_2 V_2^2}{2 h_{t2}} = \frac{p_1}{2_o C_p T_{t1} J_{t1}} + \frac{\rho_1 V_1^2}{2 h_{t1}}$$

Using Eq. (58) and the relation

$$\rho = \frac{\rho}{o\rho} \frac{p}{RT}$$

$$p_2 \left[\frac{1}{2_o C_p T_{t1} J_{t1}} + \left(\frac{\rho}{o\rho} \right)_2 \frac{X_2}{RT_2} \right] = p_1 \left[\frac{1}{2_o C_p T_{t1} J_{t1}} + \left(\frac{\rho}{o\rho} \right)_1 \frac{X_1}{RT_1} \right]$$

Using the perfect gas relation

$$R = \frac{o C_p (o\gamma - 1)}{o\gamma}$$

and solving for p_1 , we get

$$p_1 = p_2 \frac{T_1 \left[\left(\frac{\rho}{o\rho} \right)_2 X_2 T_{t1} J_{t1} + \left(\frac{o\gamma - 1}{2_o\gamma} \right) T_2 \right]}{T_2 \left[\left(\frac{\rho}{o\rho} \right)_1 X_1 T_{t1} J_{t1} + \left(\frac{o\gamma - 1}{2_o\gamma} \right) T_1 \right]} \quad (71)$$

CONCLUDING REMARKS

Numerous equations have been derived in the preceding pages. For the convenience of readers who are interested in the basic equations required for the calculation of specific gas properties, the following list is provided:

<u>Property</u>	<u>Eq.</u>	<u>Property</u>	<u>Eq.</u>
C_p	30	K	45
C_v	28	\hat{L}	61
ρ/ρ_0	7	E	44
I	51	F	38
J	47	S_0-S_1	39

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APPENDIXES

A. CONSTANTS FOR AIR

B. CALCULATION PROCEDURES

The calculation of isentropic expansions of air by the method presented is both simple and rapid. Usually the second iteration provides a sufficiently accurate result. The difference between the results obtained by using the perfect gas equations and the Beattie-Bridgeman equations is generally less than 5 percent for the range of pressures and temperatures investigated.

The calculation of air flows through normal shock waves, by the method presented, is rather tedious and time consuming if a desk calculator is used. However, most electronic digital computers can handle the problem easily if it is necessary to have a greater accuracy than provided by the perfect gas equations. The difference between the results obtained for air with the perfect gas equations and the Beattie-Bridgeman equations is generally less than 5 percent for the range of pressures and temperatures investigated.

It is interesting to note that the Beattie-Bridgeman calculations predict an increase in stagnation temperature across a strong normal shock wave. Figure 8 of Ref. 7 shows a recovery factor slightly greater than one at the stagnation point for the Mach 3.24 condition, verifying this prediction. The fact that the recovery factor decreased at Mach 4.18 and 4.92 can probably be attributed to the low accompanying p_{t1} . Note that the table of normal shock wave calculations shows a decrease in T_{t2} with decreasing p_{t1} for constant T_{t1} and p_{t1}/p_{t2} .

APPENDIX A

CONSTANTS FOR AIR

The Beattie-Bridgeman constants used in the numerical computations for air are:

$$\begin{aligned}
 A_o &= 4906.5 \frac{\text{ft}^3 \text{ lb}}{\text{lb mol in}^2} & a &= 0.30931 \frac{\text{ft}^3}{\text{lb mol}} \\
 B_o &= 0.73860 \frac{\text{ft}^3}{\text{lb mol}} & b &= -0.17636 \frac{\text{ft}^3}{\text{lb mol}} \\
 & & c &= 4.0543 \times 10^6 \frac{\text{ft}^3 \text{ } ^\circ\text{R}^2}{\text{lb mol}}
 \end{aligned}$$

Other constants used in the computations are:

$$\gamma = 1.4000 \qquad C_v = 0.17141 \frac{\text{Btu}}{\text{lb } ^\circ\text{R}}$$

$$R = 10.729 \frac{\text{ft}^3 \text{ lb/in}^2}{^\circ\text{R lb mol}} = 0.068561 \frac{\text{Btu}}{\text{lb } ^\circ\text{R}}$$

universal R

From the composition of standard air given in Ref. 3, it is noted that N_2 and O_2 are the only polyatomic molecules of significant concentration. Hence, we may consider a composition of 0.78088 N_2 and 0.2095 O_2 to determine the vibration components of the various thermodynamic quantities. Due to the form of Eq. (40) the specific heat of vibration for oxygen and nitrogen will be additive. Thus, we can write

$$C_{\text{vib air}} = 0.78088 C_{\text{vib } N_2} + 0.2095 C_{\text{vib } O_2} \qquad (72)$$

Theta may be calculated from the equation

$$\theta = \frac{h \omega_e c}{k}$$

where	h = Plancks constant	$= 6.6234 \times 10^{-27} \text{ erg sec}$
	k = Boltzmanns constant	$= 1.3803 \times 10^{-16} \text{ erg/}^\circ\text{K}$
	c = Speed of light	$= 2.9978 \times 10^{10} \text{ cm/sec}$
	ω_e = Normal mode of vibration $\div c$,	$\frac{1}{\text{cm}}$

Unfortunately, the value of ω_e for a given molecule has several values which are dependent upon the state of the molecule. However, since the temperature range covered in this work is too low to shift an appreciable proportion of the molecules into the higher electronic levels, the normal or unexcited state is used. Reference 6 gives these values as

$$\omega_{eN_2} = 2359.6 \frac{1}{\text{cm}}$$

$$\omega_{eO_2} = 1580.4 \frac{1}{\text{cm}}$$

Thus, we calculate

$$\theta_{N_2} = 6109.7 \text{ }^\circ\text{R}$$

$$\theta_{O_2} = 4092.1 \text{ }^\circ\text{R}$$

The specific heat of air due to vibration can now be calculated from Eqs. (40) and (72) using the above values of θ . Thus, we obtain

$$C_{\text{vib air}} = \left[\frac{706.85/T}{\sinh(3054.9/T)} \right]^2 + \left[\frac{245.22/T}{\sinh(2046.1/T)} \right]^2 \quad (73)$$

In a similar manner G for air is determined from Eq. (42) to be

$$G_{\text{air}} = \frac{-4770.9/T}{1 - \exp(-6109.7/T)} - \frac{857.29/T}{1 - \exp(-4092.1/T)} \quad (74)$$

Since $\ln E$ is desired for Eq. (39), we must write

$$\ln E_{\text{air}} = 0.78088 \ln E_{N_2} + 0.2095 \ln E_{O_2}$$

From this we obtain

$$E_{\text{air}} = E_{N_2}^{0.78088} E_{O_2}^{0.2095}$$

By using Eqs. (42) and (44) the final result is found to be

$$E_{\text{air}} = \left[\frac{\exp G_{N_2}}{1 - \exp(-6109.7/T)} \right]^{0.78088} \left[\frac{\exp G_{O_2}}{1 - \exp(-4092.1/T)} \right]^{0.2095} \quad (75)$$

APPENDIX B

CALCULATION PROCEDURES

ISENTROPIC EXPANSION

After an isentropic expansion from the stagnation conditions T_{t_1} and p_{t_1} to a pressure p_1 , the Mach number, M_1 , and temperature, T_1 can be obtained as follows:

- (1) Determine the perfect gas values of the Mach number, ${}_oM_1$ and temperature ${}_oT_1$.
- (2) Obtain I_1 , J_1 and K_1 for the values of p_1 and ${}_oT_1$. Obtain J_{t_1} and K_{t_1} for the values of p_{t_1} and T_{t_1} .
- (3) Solve Eq. (57) for M_1 and Eq. (55) for T_1 .

(Note: The last value obtained for a quantity should be substituted in subsequent equations which require that quantity. For example, the value of M_1 obtained with Eq. (57) should be used in Eq. (55) to calculate T_1).

- (4) Obtain new values of I_1 , J_1 , and K_1 , using p_1 and the value of T_1 , obtained from solving Eq. (55).
- (5) Solve Eq. (57) for M_1 and Eq. (55) for T_1 . Repeat this iteration process until successive approximations produce the same values of M_1 and T_1 to the accuracy desired.

CALCULATED VALUES FOR AIR

Eqs.	p_{t_1} psia	T_{t_1} °R	p_1 psia	T_1 °R	M_1
PG	2200	2200	0.150	142	8.52
BB				155	8.46
PG	2000	2200	0.100	130	8.93
BB				141	8.86
PG	2000	1500	0.750	158	6.53
BB				162	6.56
PG	2000	1000	1.50	128	5.84
BB				127	5.89
PG	1500	1000	10.0	239	3.99
BB				238	4.02

NORMAL SHOCK WAVES

With T_{t1} , p_{t1} and p_{t2} Known

When a total head tube is placed in a supersonic stream it is generally assumed that the pressure measuring instrument involved reads the total pressure, p_{t2} , behind a normal shock wave. The stagnation conditions p_{t1} and T_{t1} ahead of the shock are generally known. It is often desirable to calculate the free-stream Mach number M_1 when these three quantities are known. The following procedure will yield M_1 as well as p_1 , T_1 , p_2 , T_2 , T_{t2} , and ρ_1/ρ_2 :

- (1) Assuming $T_{t2} = T_{t1}$ and using the values of p_{t1} , and p_{t2} , determine J_{t1} and J_{t2} .
- (2) Solve Eq. (68) for T_{t2} .
- (3) Determine J_{t2} for p_{t2} and the value of T_{t2} obtained from Eq. (68).
- (4) Solve Eq. (68) again for T_{t2} . Repeat this iteration process until successive approximations of T_{t2} produce the same value to the accuracy desired.
- (5) Calculate the perfect gas values of ${}_oM_1$, ${}_oT_1$, ${}_op_1$, ${}_oT_2$, ${}_op_2$ and ${}_o(\rho_1/\rho_2)$.
- (6) Determine I_1 , J_1 , $(\rho/{}_op)_1$, for ${}_oT_1$ and ${}_op_1$; J_2 , K_2 , $(\rho/{}_op)_2$ for ${}_oT_2$ and ${}_op_2$; and K_{t2} for T_{t2} and p_{t2} .
- (7) Solve the following equations in order: Eq. (58) for X_1 , Eq. (67) for X_2 , Eq. (60) for T_1 , Eq. (60) for T_2 , Eq. (46) for p_2 where Eq. (46) is written in the form:

$$p_2 = \frac{p_{t2} K_2}{K_{t2}} \left(\frac{T_2}{T_{t2}} \right)^{\frac{o\gamma}{o\gamma-1}}$$

and Eq. (71) for p_1 .

(NOTE: The last value obtained for a quantity should be substituted in subsequent equations which require that quantity. For example, when calculating p_1 with Eq. (71) the values of X_1 , X_2 , T_1 , T_2 , and p_2 obtained in the preceding calculations should be used in Eq. (71).)

- (8) Determine I_1 , J_1 , K_1 , L_1 for the new values of T_1 and p_1 ; and J_2 , K_2 , L_2 for the new values of T_2 and p_2 .
- (9) Solve for ρ_1/ρ_2 and M_1 , respectively, from Eqs. (66) and (57).
- (10) Repeat the above procedure for X_1 , X_2 , ρ_1 , T_1 , p_1 , and T_2 . Determine new values of I_1 , J_1 , K_1 , L_1 , J_2 , K_2 , and L_2 , and solve again for ρ_1/ρ_2 and M_1 . This iteration process should be continued until all calculated values repeat to the accuracy desired.

With T_{t1} , p_{t1} , and M_1 Known

The static conditions before and after a normal shock wave and the stagnation conditions after the shock can be determined when M_1 , p_{t1} , and T_{t1} are known. The gas properties obtained during the calculation process are: p_1 , T_1 , p_2 , T_2 , p_{t2} , ${}_oT_1$, and ρ_1/ρ_2 . The calculation procedure is as follows:

- (1) Calculate the perfect gas values of ${}_op_1$, ${}_oT_1$, ${}_op_2$, ${}_oT_2$ and ${}_op_{t2}$.
- (2) Determine I_1 , J_1 , K_1 for ${}_op_1$ and ${}_oT_1$; J_{t1} , K_{t1} for p_{t1} and T_{t1} .
- (3) Solve the following equations in order: Eq. (58) for X_1 , Eq. (60) for T_1 and Eq. (46) for p_1 in the form

$$p_1 = \frac{p_{t1} K_1}{K_{t1}} \left(\frac{T_1}{T_{t1}} \right)^{\frac{o\gamma}{o\gamma-1}}$$

(Note: The last value obtained for a quantity should be substituted in subsequent equations which require that quantity. For example, when calculating Eq. (46) for p_1 , the value of T_1 obtained with Eq. (60) should be used in Eq. (46).

- (4) Determine I_1 , J_1 , K_1 for these new values of p_1 and T_1 .
- (5) Repeat the above calculation procedure for X_1 , T_1 , and p_1 . This iteration process should be continued until all calculated values repeat to the desired accuracy.
- (6) Determine L_1 for p_1 and T_1 ; J_2 , L_2 for ${}_op_2$ and ${}_oT_2$.
- (7) Solve Eq. (66) for (ρ_1/ρ_2) , Eq. (67) for X_2 , Eq. (60) for T_2 , and Eq. (70) for p_2 .
- (8) Determine J_2 , L_2 for these new values of p_2 and T_2 .
- (9) Repeat the above calculation procedure for (ρ_1/ρ_2) , X_2 , T_2 , and p_2 . This iteration process should be continued until all calculated values repeat to the accuracy desired.
- (10) Assuming ${}_oT_{t2} = T_{t1}$ determine ${}_oJ_{t2}$ and ${}_oK_{t2}$.
- (11) Solve Eq. (68) for T_{t2} and Eq. (46) for p_{t2} in the form

$$p_{t2} = \frac{p_2 K_{t2}}{K_2} \left(\frac{T_{t2}}{T_2} \right)^{\frac{o\gamma}{o\gamma-1}}$$

- (12) Determine J_{t2} and K_{t2} for these new values of p_{t2} and T_{t2} .
- (13) Repeat the above calculation procedure for T_{t2} and p_{t2} . This iteration process should be continued until all calculated values repeat to the accuracy desired.

CALCULATED VALUES

Eq.	P_{t1}	T_{t1}	P_{t2}	P_1	T_1	P_2	T_2	ρ_1/ρ_2	T_{t2}	M_1
PG*	4000	1900	40	0.519	147	35.9	1842	0.181	1900	7.71
BB**				0.508	154	36.2	1877	0.171	1925	7.78
PG	3000	1900	30	0.389	147	27.0	1842	0.181	1900	7.71
BB				0.385	155	27.1	1871	0.172	1919	7.74
PG	2500	1900	25	0.324	147	22.5	1842	0.181	1900	7.71
BB				0.323	156	22.6	1867	0.172	1915	7.72
PG	2000	1900	20	0.259	147	18.0	1842	0.181	1900	7.71
BB				0.260	156	18.1	1864	0.172	1912	7.70
PG	1500	1900	15	0.195	147	13.5	1842	0.181	1900	7.71
BB				0.195	156	13.6	1861	0.172	1909	7.68
PG	1000	1900	10	0.130	147	8.98	1842	0.181	1900	7.71
BB				0.131	157	9.04	1858	0.172	1906	7.66
PG	500	1900	5	0.0648	147	4.49	1842	0.181	1900	7.71
BB				0.0659	157	4.52	1855	0.172	1903	7.64
PG	2500	2200	7	0.0523	101	6.31	2134	0.175	2200	10.20
BB				0.0526	110	6.36	2167	0.163	2218	10.10
PG	2500	1960	11	0.0997	108	9.91	1901	0.176	1960	9.24
BB				0.0994	115	9.97	1928	0.167	1976	9.23
PG	2000	1900	15	0.171	131	13.5	1844	0.179	1900	8.22
BB				0.171	139	13.6	1865	0.170	1912	8.21
PG	1500	1900	100	3.19	328	88.8	1837	0.201	1900	4.90
BB				3.20	348	89.4	1855	0.192	1909	4.87
PG	1000	1700	50	1.40	259	44.5	1645	0.197	1700	5.28
BB				1.38	271	44.7	1657	0.189	1705	5.26

*Perfect gas equation

**Beattie-Bridgeman equation

ILLUSTRATIONS

Several of the thermodynamic properties and Beattie-Bridgeman correction factors for air have been calculated using the equations derived in the body of this report. These quantities are contained in tabular form in Ref. 8. Graphs of the tabulated values are included herein. Quick approximate solutions of various thermodynamic problems can be obtained using the graphs. A Mollier diagram for air is also included; the values of enthalpy and entropy were not included in the tables of Ref. 8.

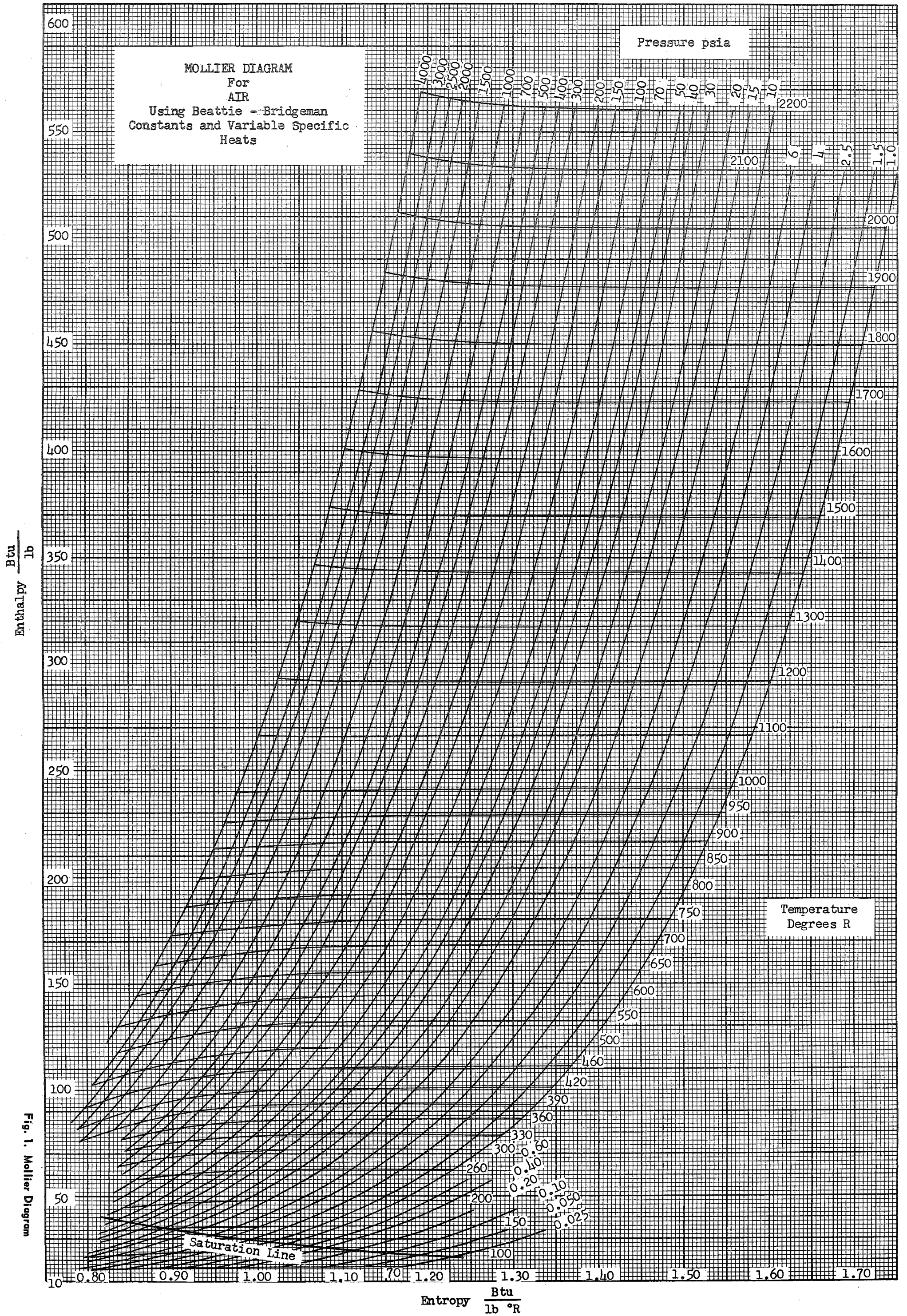


Fig. 1. Mollier Diagram

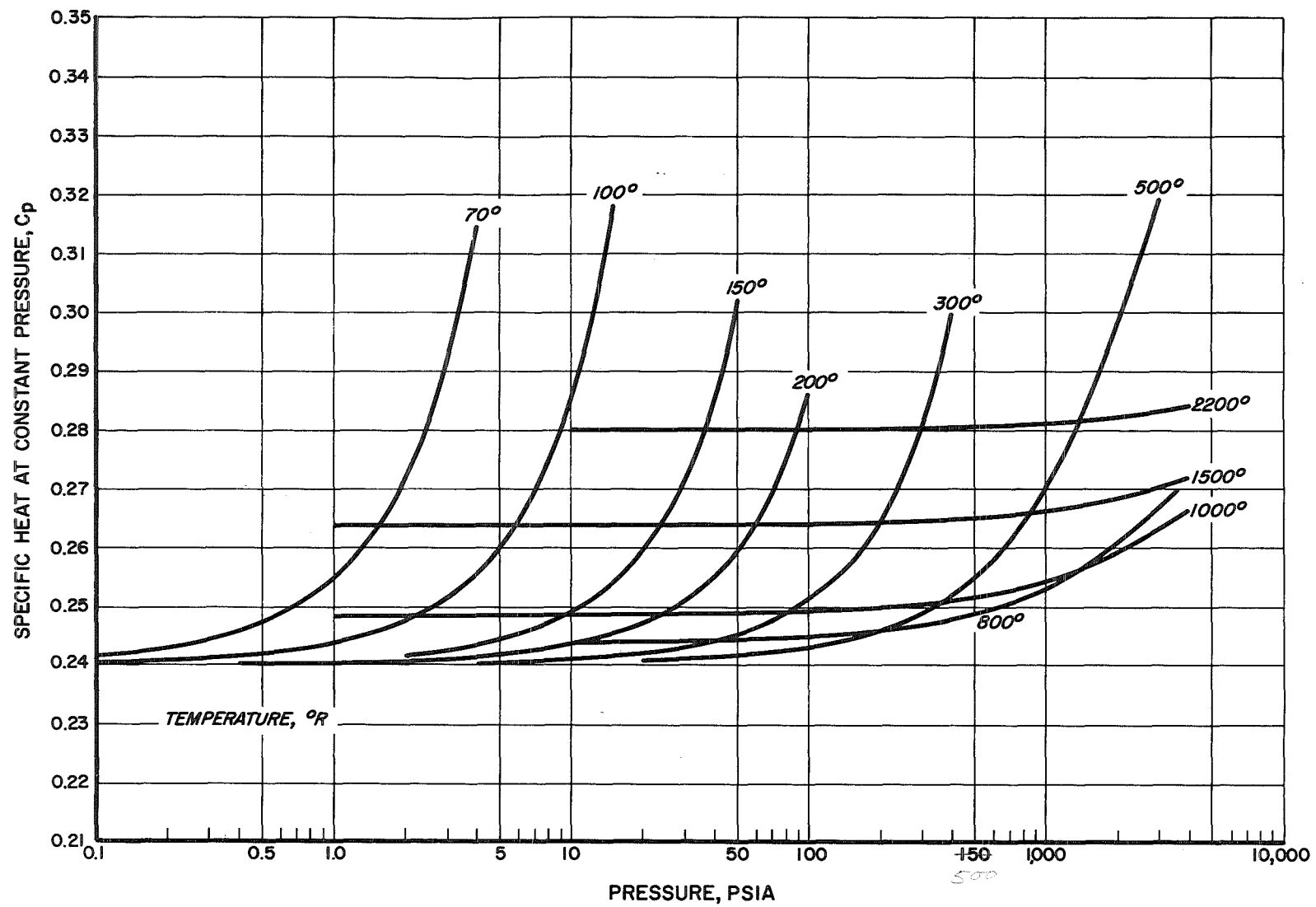


Fig. 2. Specific Heat at Constant Pressure vs Pressure

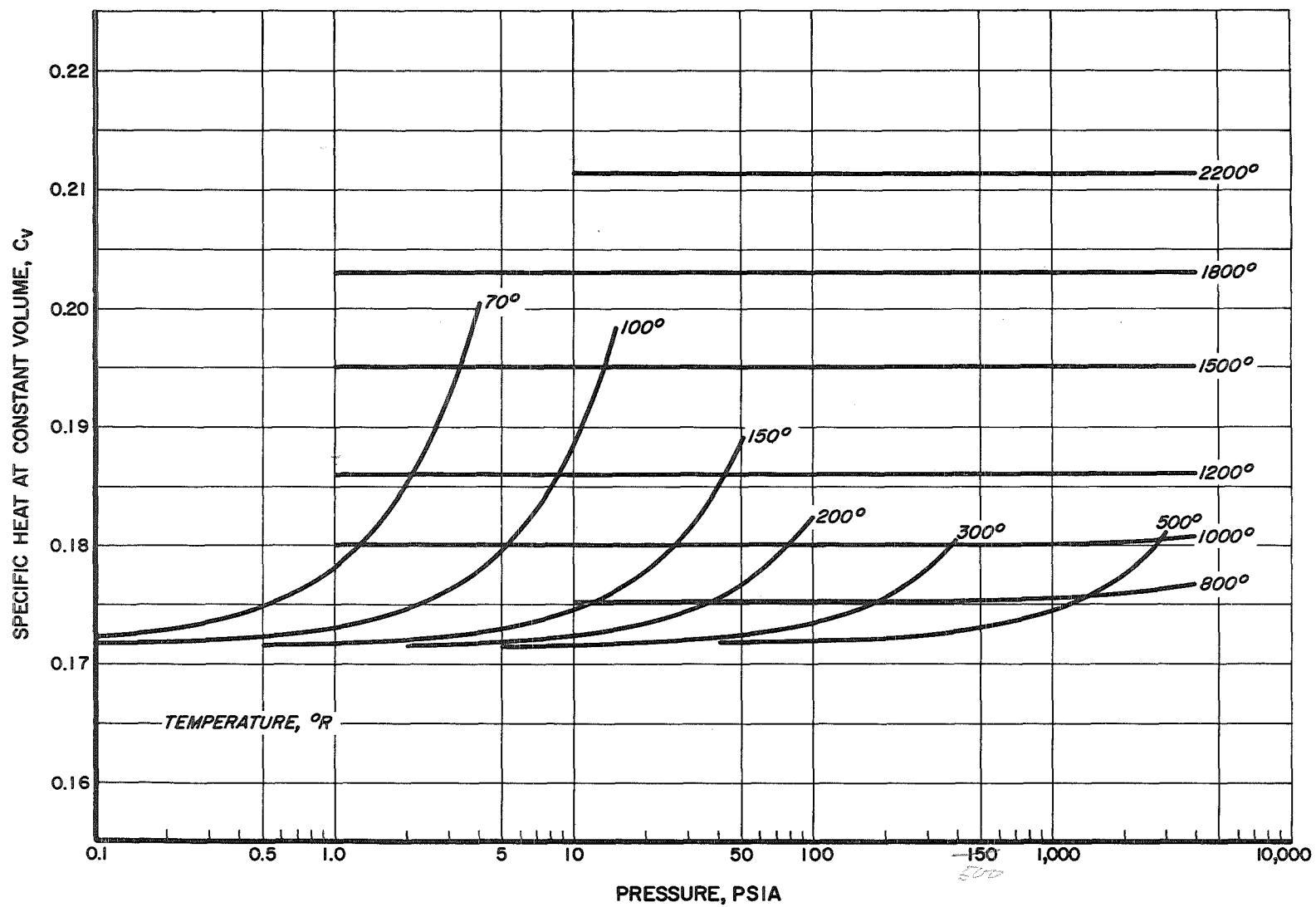


Fig. 3. Specific Heat at Constant Volume vs Pressure

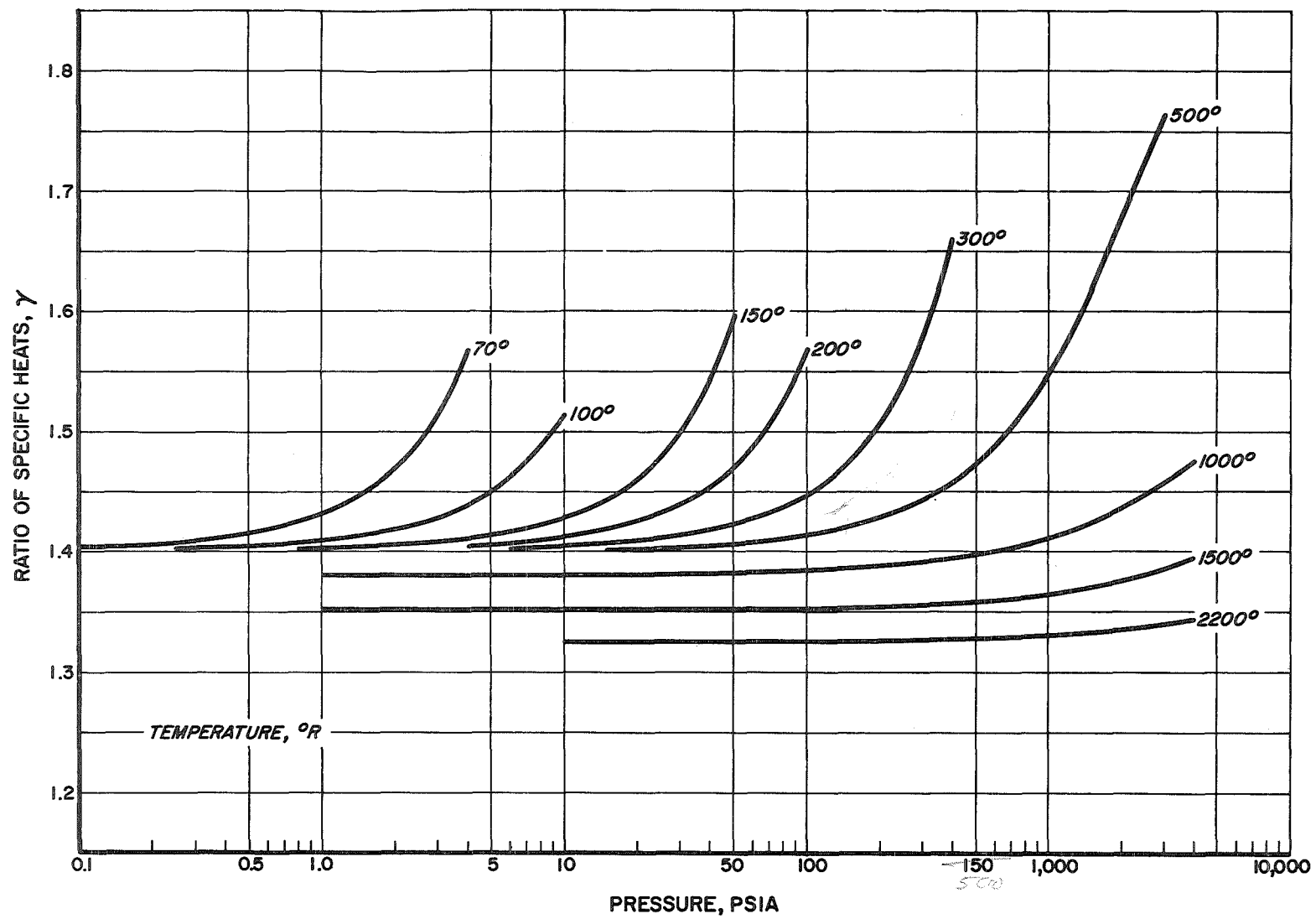


Fig. 4. Ratio of Specific Heats vs Pressure

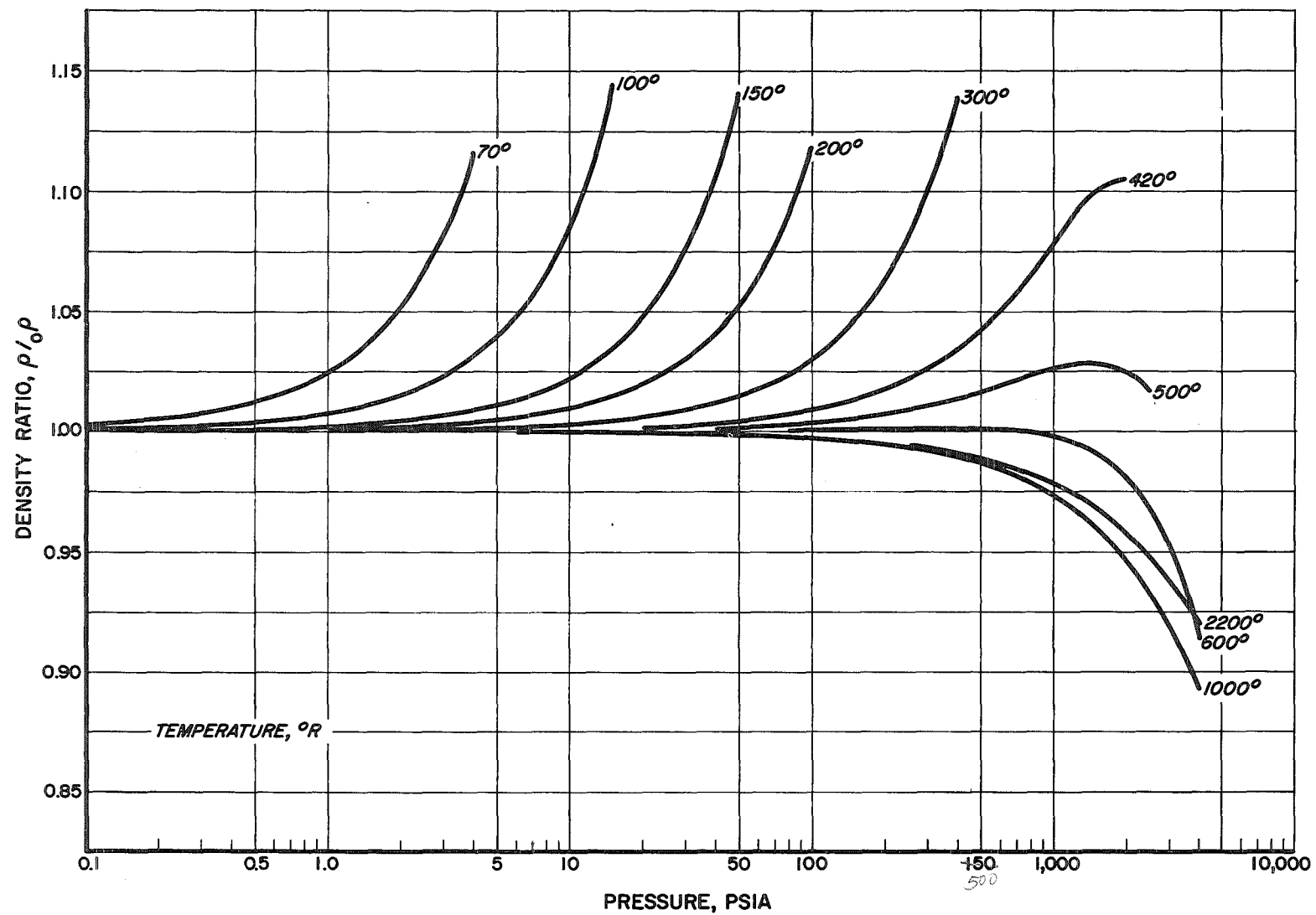


Fig. 5. Density Ratio vs Pressure

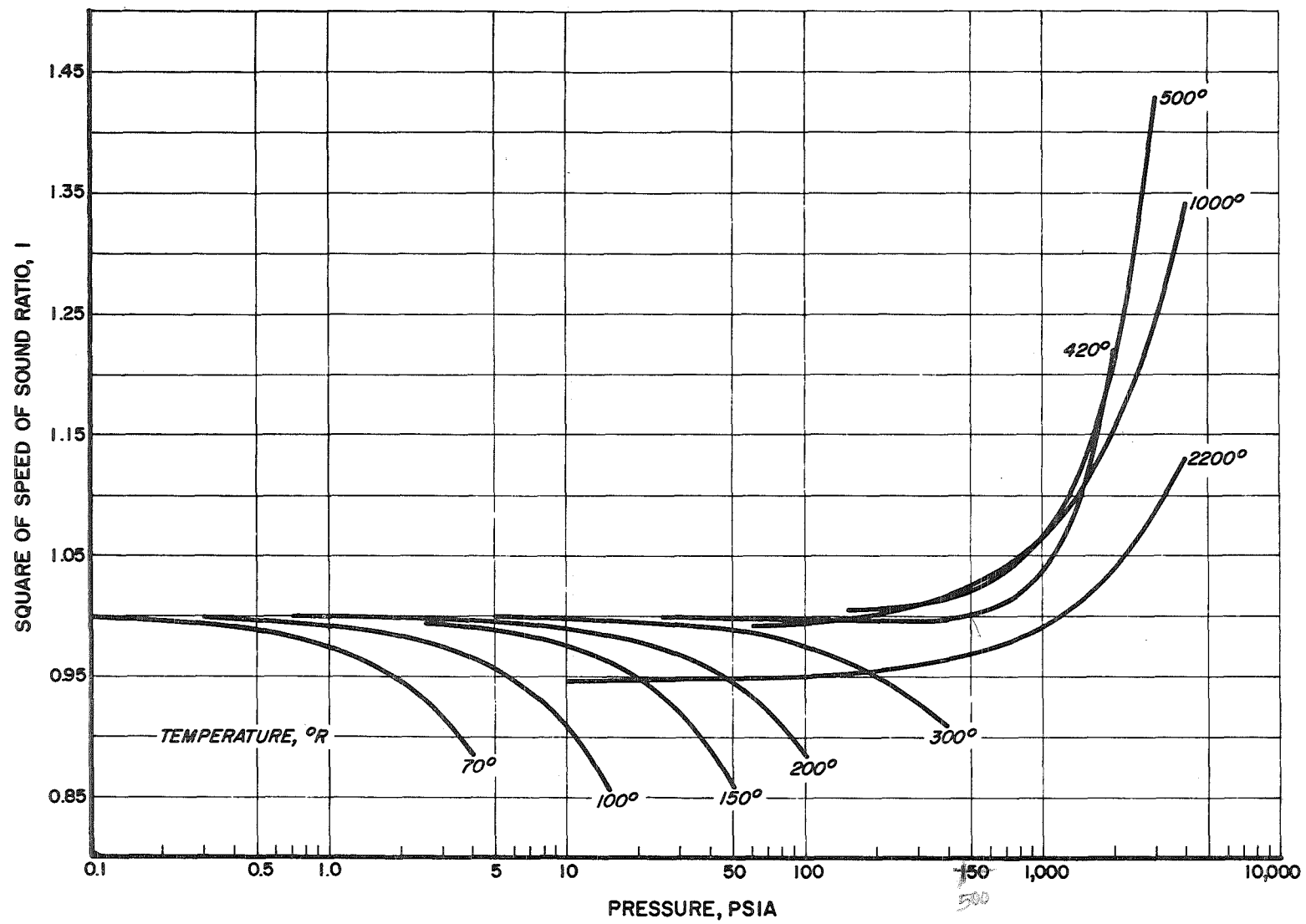


Fig. 6. Speed of Sound Ratio Squared vs Pressure

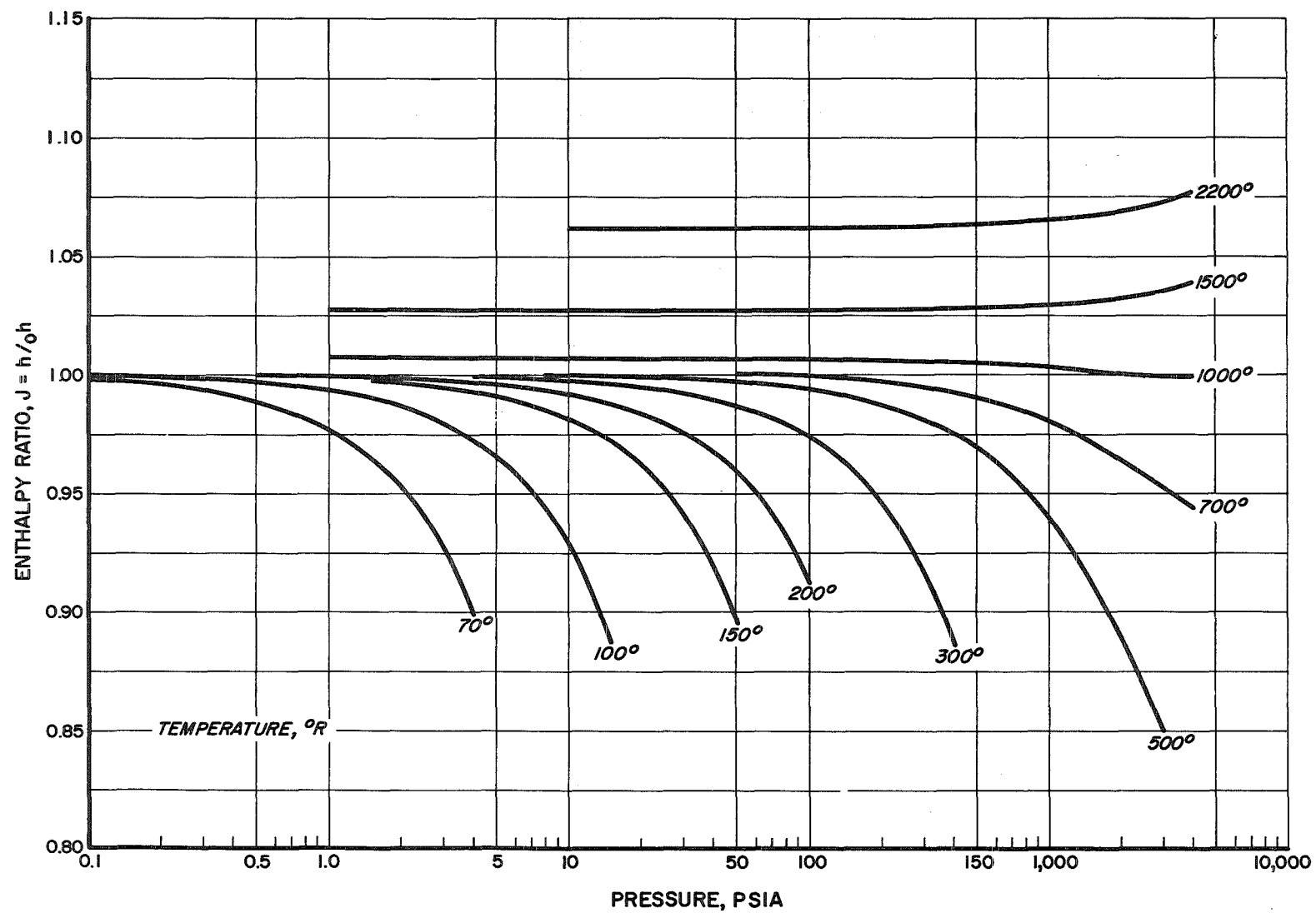


Fig. 7. Enthalpy Ratio vs Pressure

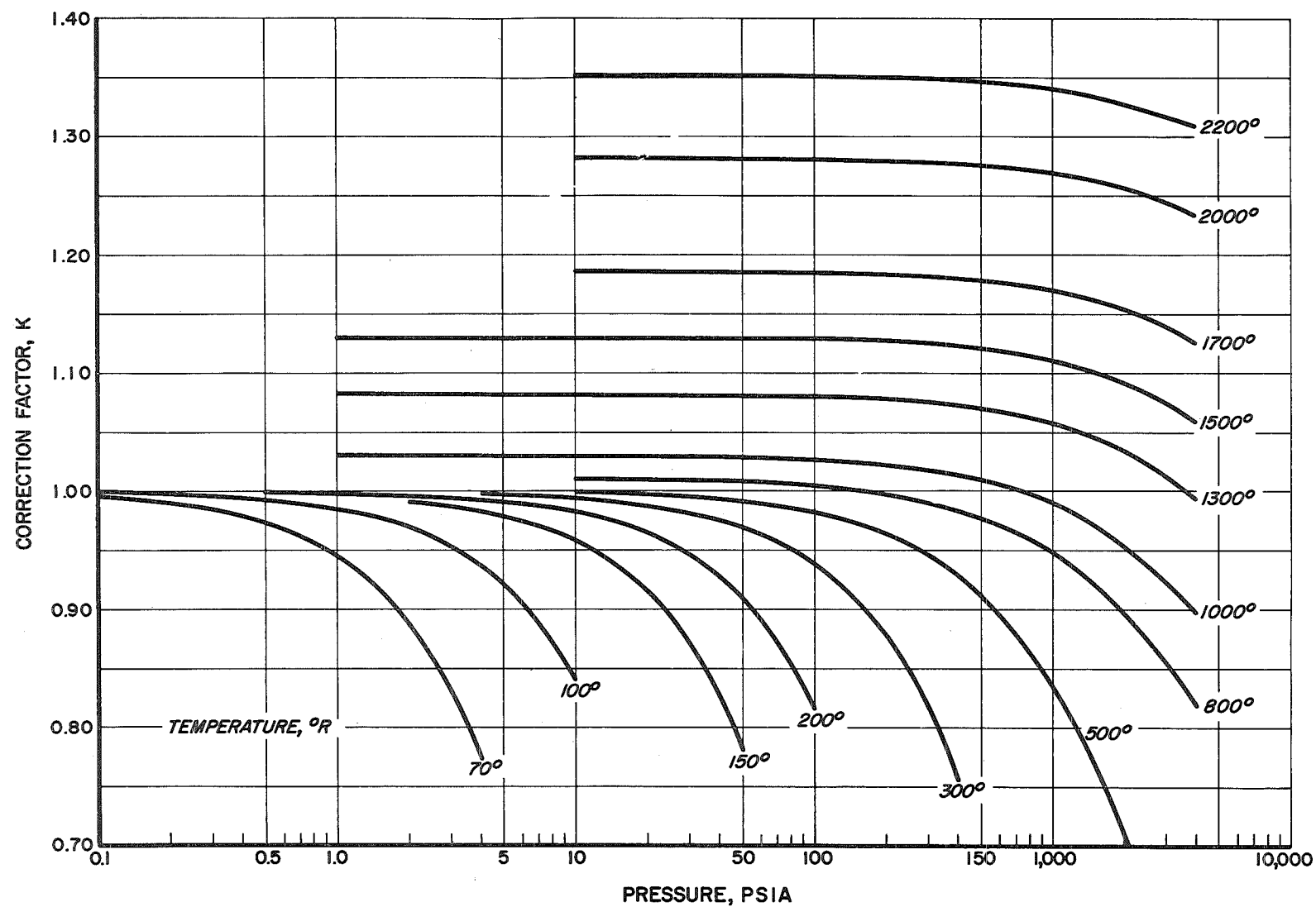


Fig. 8. Correction Factor K vs Pressure

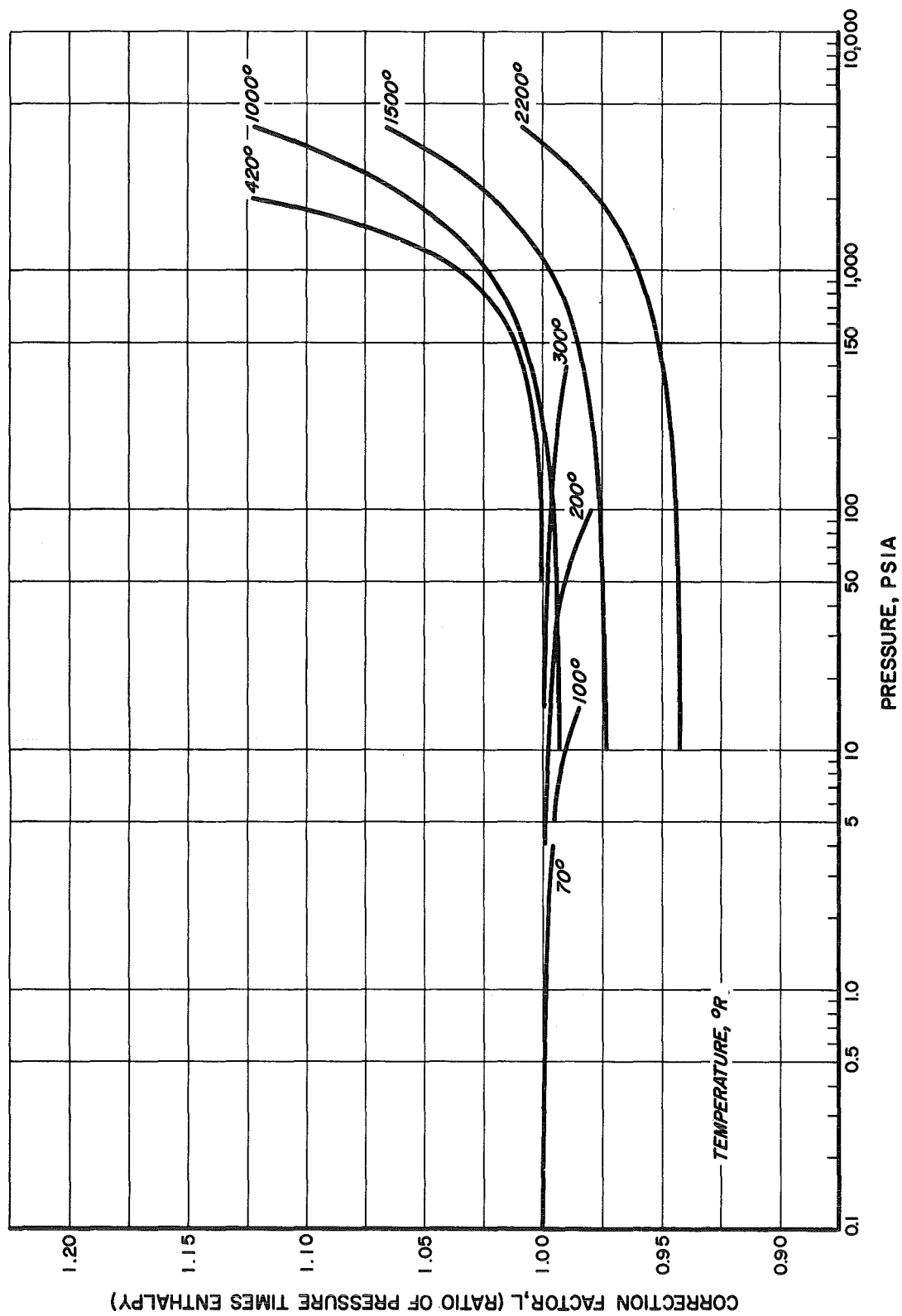


Fig. 9. Correction Factor L vs Pressure

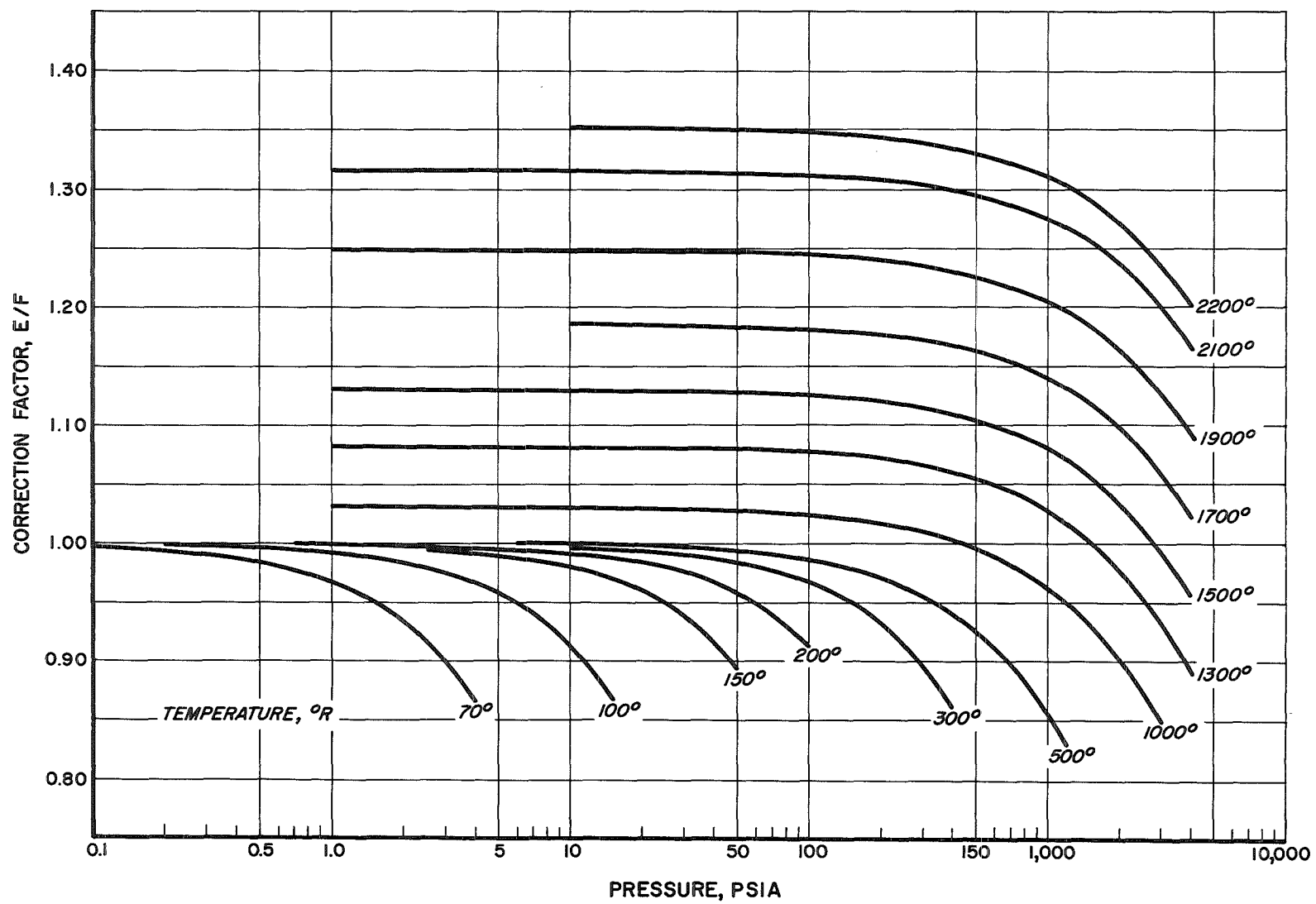


Fig. 10. Correction Factor E/F vs Pressure

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Investigated were isentropic expansion and flow through normal shock waves. The calculation procedures for isentropic expansions and flow through normal shock waves are included in the appendix. Also included are the results of several calculations in which air was used as the media of flow. Graphs of the thermodynamic properties and Beattie-Bridgeman correction factors for air are included to provide a general picture of the effect of temperature and pressure.

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THERMODYNAMIC PROPERTIES OF GASES;
EQUATIONS DERIVED FROM THE BEATTIE-
BRIDGEMAN EQUATION OF STATE ASSUMING
VARIABLE SPECIFIC HEATS by R. E. Randall,
August 1957. 44 pp. (Contract No. AF 40(600)-700
Sup. 6(58-1)

8 references

The Beattie-Bridgeman equation of state was used to develop the equations of several of the thermodynamic properties and flow process correction factors for gases. The increase in the specific heats due to the vibration of diatomic molecules was included by assuming the molecules to be perfect harmonic oscillators. Thermodynamic and flow process equations are theoretically developed. The particular flow processes

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 3. Gases--Specific heat (Variable)
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